# The state properties of adsorbed fluids 

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#### Abstract

The fundamental equation for the pressure of adsorbed fluids, containing the potential energy of the adsorbed moleoule as an parameter, has been derived on the besis of the thermodynamics of fiuids in the field of adsorption forces. As an example of its applicability, the state function of krypton adsorbed on the zeolite CaA has been evaluated. This fumotion substantially deviates from that of the bulk krypton as a result of the smaller number of interacting atoms in the small volumes of adsorbate. On the basis of the state equations of adsorbed fluids, the equations of adsorption isotherms for energetically homogeneous adsorbents are derived. From this equations follows, that the shape of the isotherms in the ooordinates amount adsorbed vs. 10garithm of the equilibrious gas pressure, is determined by the state equation whereas its position on the absoissae axis depends on the potential energy of the adsorbed molecule in the adsorption space.


## INTRODUCTION

The phenomenon of physical adsorption is caused by the enhanoement of the density and concentration of fluids on the surfaces of solids or in micropores of solids, as a direct consequence of the existence of the attractive adsorption forces. This density is determined by the $p^{2}, v^{a}, T$ state relation of a given adsorbed fluid. The pressure $p^{2}$ of this fluid depends on the pressure $p^{g}$ of the equilibrious gas existing out side the influence of adsorption forces and on the value of the potential of theseforces, further denoted as $\varnothing$. The relation between the above pressures and the potential $\varnothing$ can be evaluated by means of the general thermodynamical condition for equilibrium of fluids in the field of external forces with respeot to the transport of matter firstly formulated by J.W.Gibbs. From the above ideas it is evident, that the theory of physical adsorption may be olosely connected with the theory of $s$ tate properties of fluids and eventually also with the extra complications in these properties oaused by the presence of the surface of solid.

In the desoribed treatment it is important to choose properiy the concept of the structure of the adsorbed fluid. When the adsorbed fluid forms only a monolayer on the solid surface, the concept of the two-dimensional (2D) fluid ahould be chosen. The properties of this fluid can be desoribed by 2D state functions suah as the surface pressure $\pi$ and the molar surface $\sigma$ of the adsorbed fluid, eto. But when adjacently adsorbed moleoules can take arbitrary positions in three dimensions with respect to each other, the three-dimensignal (3D) oonoept of adsorbed fiuid is to be applied and the 3D pressure $p$ and the 3D molar volume $v$ of the adsorbate should be introduced. The latter case concerns e.g. the adsorption of vapours in microporous solids, whioh will be mainly discussed in the present paper. There is an interesting correspondence between both concepts. As has been shown by E.A.FIood (ref. 1), they can be easily transformed into one another by a smail change in the language.

Because in our treatment, the thermodynamios in the field of extermal foroes is applied, the question of its applicability to adsorption phenomena should be elucidated. The basic assumption is, that the adsorbate behaves as a single component substance, remaining distinct and ocoupying separate and definite volumes (raf. 2). Than the important limitation of the scope of the Second Law of thermodynamics in small regions of colloidal systems
can be overcomed by the interpretation of the state $f$ unctions, such as pressure, temperature, volume, chemical potential etc., as time averaged quantities. Thus e.g. the physical meaning of the pressure $p^{\text {a }}$ is the averaged force per unit area, which the moleoules of the adsorbate on one side of the infinitesimal surface element exert on the other side. This averaging is clearly appropriate because of the macroscopic nature of the thermodynamic pressure, in contrast to the moleoular dimensions of the surfaoe regions. As a consequence, the Gibbsian condition for the equilibrium of the fluid, for every point of the adsorption space, where the potential energy of the adsorbed molecule is $\varnothing$, may be written in the form

$$
\begin{equation*}
\mu^{a}\left(p^{a}, T^{a}\right)+\varnothing=\mu^{g}\left(p^{g}, T\right) \quad, \quad T^{a}=T=\text { const } . \tag{1}
\end{equation*}
$$

where the state functions $\mu^{a}, p^{a}, T^{a}\left(\mu^{a}\right.$ is the ohemical potential of adsorbed fluid and $T^{2}$ is the temperature) represents the time averaged quantities. $\mu^{8}$ is the chemical potential of the equilibrious gas outside the influence of adsorption forces in the space, fhere the potential $\varnothing$ is zero. The relation between the pressures $p^{a}$ and $p^{8}$ of the adsorbed and equilibrious gas, respeotively, important for the theory of physical adsorption, follows from the Gibbsian condition of equilibrium, as is olerly seen from its form (1), similarly as the relation between the surface pressure $\pi$ and the gas pressure $p^{E}$ oan be evalugted from the Gibbsian adsorption equation. The explicit form of this $p^{a}\left(\varnothing, p^{b}\right)$ function is derived and its application in the analysis of the state properties of adsorbed fluids is demonstrated in the following chapters.

## THE EQUATION FOR THE PRESSURE OF ADSORBED FLUIDS

As hes been disoussed above, the equation for the evaluation of the pressure $p^{a}$ from the adsorption isotherm can be derived on the basis of equation (1). The chemical potentials $\mu^{2}$ and $\mu^{B}$, whioh appears in this equation, depends on the corresponding pressures of the adsorbed and equilibrious gas. This dependence for the ohemical potential of the liquid-like adsorbed fluid can be written in the form

$$
\begin{equation*}
\mu^{a}=\mu_{0}^{1}+\int_{p_{0}^{1}}^{p^{a}} v^{a} d p^{a}+\mu^{\mathbf{x}} \quad, \quad \text { T=const. } \tag{2}
\end{equation*}
$$

where $\mu^{1}$ and $p_{0}^{1}$ are the chemical potential and pressure, respectively, of the bulf 1iquid in the standard state at the temperature $T$ and $\mu$ is the change in the chemical potential of the bylk liquid, when it is transferred in the adsorption space at the pressure $p_{0}$. The chemioal potential of equilibrious vapour at given temperature $T$ is ${ }^{\circ}$

$$
\begin{equation*}
\mu^{G}=\mu_{0}^{G}+\int_{p_{0}^{g}}^{p^{G}} v^{g} d p^{g}=\mu_{0}^{g}+R T \ln x+B\left(p_{o}^{g}-p^{g}\right)+\ldots \tag{3}
\end{equation*}
$$

where $\mu_{0}^{S}$ is the ohemioal potential of the vapour in the standard state, when its pressure is $p_{0}^{E}, x=p^{8} / p_{0}^{E}$ is the relative pressure of equilibrious vapour and $B$ is the second virial coefficient of the Berliner form of the virial gas state equation.

Now we try to evaluate the molar volume $\mathrm{v}^{\mathrm{a}}$ of adsorbed fluid by means of the adsorbed amount $a(x)$. When only the liquid-like or the gas-like fluid fill the volume $V_{0}$ of the adsorption space; in whioh the potential energy $\varnothing$ of the adsorbed molecule has a constant value, the molar volume $\nabla^{\circ}$ of this mobile fluid can be evaluated by the relation

$$
\begin{equation*}
v^{a}=v_{0} / a(x) \tag{4}
\end{equation*}
$$

where $a(x)$ represents the so called full amount adsorbed at relative pressure $x$ and is related to the experimentally meassurable adsorbed amount $a_{e x}(x)$ by the relation

$$
\begin{equation*}
a(x)=a_{e x}(x)+\left(p^{g} V_{0}\right) / R T, \tag{5}
\end{equation*}
$$

Here it is important to emphasize, that the relation (4) is not generally valid. When the volupe $V$ is partially filled by the adsorbed liquid-like fluid at negative $p$ pressure and at the same time partially by the gas-
like fluid of positive $p^{\text {ag }}$ pressure, this relation is not valid. In this case eqn. (4) gannot be used for the evaluation of the value of the chemioal potential $\mu^{a}$ according the equation (2).

When we choose for convenience as standard that state of bulk vapour and liquid, where are in thermodynapioal and qeohanical equilibrium, when separated with flat interface ( $\mu_{0}^{\delta}=\mu_{0}$ and $p_{0}^{g}=p_{0}^{1}$ ), then if the $3 D$ vapour behaves as an ideal gas, by substituifing equailiong (2)-(4) into the equilibrium condition (1) and by rearranging the obtained expression, the fumamental equation for the pressure $p$ follows

$$
\begin{equation*}
p^{a}=\frac{R T}{V_{0}} \int_{\ln z}^{\ln x} a(x) d \ln x+p_{0}^{1} \quad, T=\text { oonst. } \tag{6}
\end{equation*}
$$

where $p_{0}^{1}$ is the pressure of the bulk liquid in the standard state, equal to the norinal saturated rapour pressure $p_{0}^{g}$ at given temperature $T$ and $z=\exp \left[\left(\phi+\mu^{z}\right) / R T\right]$ represents the relatiqe pressure of equilibrious vapour, af which the adsorbed fluid pressure $p^{a}$ is equal to the standard pressure $p_{0}$.
The equation (6) seems to be the 3D analog of the Gibbsian adsorption equation integrated according to Bangham (ref. 3)

$$
\begin{equation*}
\pi=\frac{R T}{S} \int_{\ln x=-\infty}^{\ln x} a(x) d \ln x \quad, T=\text { onst } \tag{7}
\end{equation*}
$$

where $S$ is the specific surface of the adsorbent. It is frequently used for the evaluation of the surface pressure $\pi$. Moreover equation (6) contains the potential energy $\varnothing$ of the adsorbed molecule as parameter. This is connected with the reality, that equation (1) represents the condition of eqilibrium containing ohemioal potentials in the integrated form. When the differential forms of the equilibrium cgndition are used in the evaluations of adsorbed fluid pressures (such as $d \mu^{9}=d \mu^{5}$, of. ref. 2), the potential energy $\varnothing$ is included in the integration constant, which dissapears e.g. in the Bangham's treatment, in which it is assumed that the surface pressure is zero when the equilibrious gas pressure approach to zero.

When in an analogous derivation as presented here one chooses the arbitrary low pressure $p_{+}$of the gas as standard, one obtains for the pressure $p^{a}$ the expression

$$
\begin{equation*}
p^{a}=\frac{R T}{V_{0}} \int_{\ln \left(z p_{+}^{E}\right)}^{\ln p^{G}} a\left(p^{E}\right) d l n p^{g}+p_{+}^{G}, \quad T=\text { oonst. }, \tag{8}
\end{equation*}
$$

which for $p_{t}^{g} \rightarrow 0$ yields the $3 D$ analog of the integrated Gibbsian adsorption equation

$$
\begin{equation*}
p^{a}=\frac{R T}{V_{0}} \int_{\ln x=-\infty}^{\ln x} a(x) d \ln x \quad, T=c o n s t \tag{9}
\end{equation*}
$$

where $z=\exp \left[\left(\phi_{+} \mu^{x}\right) / R T\right]$ and $\mu^{X}$ represent the change in the chemical potential of the gas, when it is transferred in the adsorption space at the pressure $p_{+}^{\text {. }}$.
The fundamental equation (6) for the pressure of the adsorbed fluid is consistent with the known relations valid in special situations. When the depsity of the adsorbed liquid does notadepend appreaiably on the pressure $p^{a}$, the mean value of the molar volume $\vec{v}^{a l}$ of the adsorbed liquid can be introduoed_in the refation (4). This relation when substituted in the rearranged form $\bar{a}(x)=V / \bar{v}$ in (6) yields the approximate expression for the pressure of the adsorbed liquid (of. ref. 4)

$$
\begin{equation*}
p^{a l}=\left[(R T) / \bar{v}^{a l}\right] \ln (x / z)+p_{0}^{1}, T=\text { oonst. . } \tag{10}
\end{equation*}
$$

The potential energy $\emptyset$ in mesopores as well as $\mu$ are zero. Hence, $z=1$ and the equation (10) is reduced to the well known expresion for $p$ from the


When we know the state equation of the adsorbed fiuid, the pressure $p$ an be evaluated without the knowledge of adsorption isotherm. The integral in the expression (2) for $\mu^{a}$ can be evaluated by means of this state function and by starting from eqns. (1)-(3) the explicit expression for $p(\phi, x)$, ontaining states parameters only, cen be derived (ref. 5).

## LIMITS OF APPLICABILITY OF THE EQUATIONS FOR THE PRESSURE $p^{a}$

It is important to realize that the equations (6), (8) and (9) oannot b successfully applied for evaluations of the pressures of adsorbed fluids from adsorption isotherms corresponding to adsorption prooesses in whlch the "phase transitions" with the metastable liquid-like fluid appear. This can be clearly demonstrated for the pressure p evaluated from the capillary condensation isotherm of vapour on a solid having oylindrical pores, al. 1 of radius $r$, open at one end, diagramatically shown in Fig. 1.


Fig. 1. Part a) represents the capillary condensation at the pressure $x=x_{\text {f }}$ and part b) represents the copression of condensed liquid correspopiting to the increase of the negative pressure $p$, when the pressure $x$ inoreases from $x=x_{\text {f }}$ to $x=1$. According to equation (6) for $x>x_{f}$, the evaluated pressure $p$ yields a correct nfgative value approximatefy given by the equation
$p^{2}=[(R T) / \bar{v}]$ inx , in agreement with the theory of capillary condensation. The reason is, that in the pressure range $x_{f}>x>1$, no phase transition occurs, i.e. pores are still volume filled by the liquid-like fluid.

On the other hand the application of equation (9) yields erroneously the positive pressure $p^{2}$ for the equilibrious relative pressure $x=1$. It is because in the gas pressure region from $x=0$ to $x=1$, the condensation of the gas-like to the liquid-like adsorbed fluid appears, in which the capillaries are filled by the condensed liquid existing in the metastable state ( $\mathbf{p}<0$ ). It is evident that similar limitation of the applicability of the integrated Gibbsian adsorption equation (7) has to be taken into aocount, if the 2D metastable fluid can exist in the adsorption process. By not respeoting the limits of applicability of the equation (4), or its 2 D analog $(\sigma=S / a(x))$ the "effective character" of the evaluated equations of states of adsorbed fluids, desoribed in the literature, oan emerge. It should be also emphasized that all the equations (6)-(9) give correct values of the adsorbed fluid pressures only in cases, when the isotherms $a(x)$ correspond to an energetically homogeneous adsorbents. This is offen not taken into account by many autors.

## THE STATE RELATIONS OF ADSORBED FLUIDS

Equations (4), (6) and (9) enable to evaluate $p^{a}, v^{a}, T$ state relations of the adsorbed fluid from adsorption isotherms. Such an evaluation for krypton is presented here as an example. The initial data taken from the paper of B.P. Bering at all (ref. 6) are redrawn in Fig. 2.


As follows from Fig. 2a., at the RTlax valueg ranging from -15 to -7.2 kJ mole ${ }^{-1}$, the inequality $\Delta 1 / 1<0$ holds. Thus $p^{\text {ail }}<0$ and henoe the adsorbed liquid-like krypton is in the metastable state. For this reason two different forms (6) and (9) of the equation for the pressure $p$ have to be applied. When the adsorption volume $V$ is ogmpletely volume filled by the li-quid-like fluid, the rigoŗous valus of $p^{a}$ can be ovaluated only by the eqn. (6). The value $V_{0}=0.325 \mathrm{~cm} / \mathrm{g}$ of the miorocavities volume (ref. 7) and the value $z$ given by the relation RTlnz=-7.2 $\mathrm{kJ} /$ mole corresponding to $\Delta 1 / 1=0$ have been used in this evaluation together with the assumption that the potential energy $\varnothing$ of krypton atoms in the volume $V_{0}$ is constant. Obviously this assumption is fulfilled only approximately. Ohen at the low gas pressures $x$, the density of adsorbed krypton deoreases so much, that it is reversibly converted into the gas-like state, equation (6) turns out not to be valid any longer. When the evaluation negleots this reality, the obtained state function has an effective oharaoter only. This is olearly seen from Fig. 3, where the real and the offeotive parts of the state funation, evaluated by means of eqn. (4) and (6), are depioted by the full and the dotted curve 1 , respectively, (the pressure of the gas-like fiuid cannot be negative). On the other hand, in the range of small adsorbed amounts $a(x)$, the pressure $p$ can be determined by eqn. (9) only. At higher adsorbed amounts, the evaluated state function again acquires the effective oharacter in spite of the presence of the "phase transitions" with the presence of the metastable liquid-like fluid. This is illustrated in Fig. 3 by the full and dgtted parts of the curve ?. The shape of the evaluated real state function $p\left(v^{2}\right)$ reminds the van der Waals state function. The hypothetioal part with the positive slope $\left(\delta \mathrm{p}^{2} / \delta \mathrm{v}^{\mathrm{a}}\right.$ ) T , joining both real branches of the state fumotion, has been estimated by means of the Maxwell rule (ourve 3).


Fig. 3. The state $p^{a}\left(v^{a}\right)$ funotion of krypton adsorbed at 180 K on the zeolite CaA, evaluated by means of the eqns. (4), (6) and (9) from the experimental data shown on Fig. 2. The evaluated function strongly deviates from that for the bulk. The bulk standard state is denoted by 0 .


Fig. 4. the relation between pressure $p$ and proper volume $v$ of fluid below the oritical tempera. ture. The reversible "condensation" $V-L, V^{\prime \prime}-L$ 'and $V^{\prime \prime}-L^{\prime \prime}$ is shown diagrammatioally.

Next we turn our attention to the meohanism of the transitions of gasmike to liquid-like adsorbed fluids. Because the mioropores of zeolite çad are volume filled by the metastable liquid-like krypton at negative $p$, this type of filling belongs to the second type of these mechanism (MVFM 2), the classification of which has been proposed recently (ref. 4). Here, the stress of the adsorbed liquid is caused by the bridgging of the opposite wals of the zeolite cavities by the krypton liquid-like clusters and have nothing common with the classioal theory of capillarity, because the surface of tension cannot be realized in the mioropores. But when we overlook the presence of the field of adsorption forces, this process is formally analogous with reversible capillary condensation in one-side open
capillaries, discussed ebove, The "oondensation" of such type can be illum strated diagramatically on Fig. 4 as the transition from the atate $V$ to the state $L^{\prime}$ on the $p, v, T$ diagram (for simplicity it is assumed that the $p\left(v^{a}, T\right)$ function is the same). This transition differs from the normal bulk fluid condensation under the equal vapour and liquid pressure ( $p_{0}^{5}=p_{0}^{1}$ ), illustrated as the transition V - L. The reversible transition of ine described type (e.g. $V^{\prime \prime}-L^{\prime}$ ) oanses the effective character of the state fumotions, evaluated from the adsorption isotherms, when the adsorption space is filied according the MVFM 2.

## THE CHANGES OF THE STATE EQUATION OF THE MATERIAL IN THE SMALL VOLUME OR LAYER OF THE ADSORBATE

As evident from Fig. 3, the state equation of adsorbed krypton oontains another parameters, than that of the bulk fluid. The above conclusion is also supported by another result. The evaluation of the potential energy $\phi$ of the adsorbate may be performed by means of the rearranged Boltzmann function $\emptyset=R T \ln \left(p^{a g} / p^{s}\right)$, where the pressure $p^{\text {ag }}$, corresponding to given $p^{g}$ is evaluated from the low pressure region of the adsorption isotherm by means of equation (9). The value $\varnothing$ for krypton on the zeolite CaA evaluated in this way is $-10.6 \mathrm{~kJ} /$ mole. On the other hand ope has RT1nz= $\varnothing+\mu=-7.2$ $\mathrm{kJ} / \mathrm{mole}$. As follows from these data, the value $\mu=3.4 \mathrm{~kJ} / \mathrm{mole}$ of the perturbed chemical potential supports the idea, that the state equation of krypton largely deviates from that of time bulk fluid. As shown by A.W. Adamson (ref. 8), the perturbed chemicel potential of the liquid-like fluid can be characterized by the relation $\mu^{*}=\operatorname{RTln}\left(p_{\omega} / p_{o}^{\delta}\right)$, where $p_{\omega}$ is the hypothetical pressure over the adsorbed fluid, remote from the field of forces, but got otherwise changed (its pressure $p^{a}=p_{0}$ ). From the determined value of $\mu$ it follows that the hypothetical pressire $p_{\omega}$ is approximately ten times higher than the standard pressure of the bulk krypton. This result agrees well with the theoretical conolusions (ref, 9). The saturated pres. sure $p_{\omega}$ of the vapour, which is in equiliprium with the liquid-like clus. ter of krypton atoms under the pressure $p_{p}$, is higher than the saturated pressure pof the bulk liquid in spite of the smaller number of interacting krypton atoms.

The smaller number of interaoting atoms or molecules in the small volume of the adsorbate is the reason e.g. of the smaller value of the constant $\alpha$ of the van der Waals equation, as evident from the expressions that follow for the bulk and adsorbed van der Waals fluid (eqns.(11) and (12), respectively)

$$
\begin{equation*}
\alpha=\frac{\mathbb{N}^{2}}{2} \int_{d}^{\infty} \frac{C_{A}}{r^{6}} 4 \pi r^{2} d r, \quad \quad(11) \quad=\frac{N^{2}}{2} \int_{d}^{D} \frac{C_{A}}{r^{6}} 4 \pi r^{2} d r \tag{12}
\end{equation*}
$$

Here $N$ is the Avogadro number, $C_{A}$ the Kirkwood-MGiler constant, diameter of the atom or the molecule and $D$ is the diameter of the mioropore oavity. The equation (12) is approximately valid, when the Law of intermolecular forces is the same in both oases (cf. ref.10,11). The critical temperature of fluids adsorbed in mioropores is olearly substantially lower than the critical temperature of the bulk fluid, quite analogously to the case of 2D fluids.

## ADSORPTION ISOTHERMS OF VAPOURS ON HOMOGENEOUS ADSORBENTS

When the relation (4) is quite correct and the equation of state of the adsorbed fluid is know, the equation of the adsorption isotherm oan be evaluated on the basis of equations (1)-(4). Here the molar volume $\mathbf{v}^{\mathbf{a}}$ of adsorbed fluid has to be the real state funotion and not a mean value coresponding to different fluid states in the separate parts of the adsorption volume $V$. Further important assumption is, that the potential energy $\varnothing$ of the m8lecule is constant in any point of the adsorption space. For these conditions the derivation of equations of adsorption isotherms for the main state equations is derived. When we again choose the state of the bulk liquid and normal saturated vapour at given temperature $T$ as standard and when we assume that the equilibrious gas behaves ideally, than the conditi-
on (1) of the equilibritu can be expressed as follows

$$
\begin{equation*}
\int_{p_{0}}^{p^{a}} v^{a} d p^{a}+\varnothing+\mu^{\pi}=R T \operatorname{lnx} \quad \text {,T=const. } \tag{13}
\end{equation*}
$$

because $\mu_{0}^{1}=\mu_{0}^{8}$ and $p_{0}^{1}=p_{0}^{E}$ holds. The integral on 1.h.s. of this equation can be evaluated ${ }^{\circ}$ by means of the equations of state. The obtained $x\left(v^{a}, \phi\right)$ function can be transformed in the equation of adsorption isotherm by means of the relation (4).
(i) The virial adsorption isotherm

Now consider that the Leyden form of the virial state equation

$$
\begin{equation*}
\frac{p^{a} v^{a}}{R T}=1+\frac{A}{v^{a}}+\frac{B}{\left(v^{a}\right)^{2}}+\frac{c}{\left(v^{a}\right)^{3}}+\cdots \tag{14}
\end{equation*}
$$

describes well the $\mathrm{p}^{\mathrm{a}}, \mathrm{v}^{\mathrm{a}}, \mathrm{T}$ state relations of the adsorbed fluid. The evaIuation of the integral of equation (13) by means of (14) may be performed per partes

$$
\begin{equation*}
\int_{p_{0}^{1}}^{p^{a}} v^{a} d p^{a}=\int_{p_{0}^{1} v_{0}^{1}}^{p^{a} v^{a}} d^{a}\left(p^{a}\right)-\int_{v_{1}}^{v^{a}} p^{a} d v^{a} \tag{15}
\end{equation*}
$$

where $v^{l}$ is the molar volume of the bulk liquid in the standard state. From eqgs. (13)-(15) the followine equation of adsorption isotherm follows

$$
\begin{equation*}
x=\exp \left[\left(\phi+\mu^{x}\right) / R T\right] \theta \exp \left[K+\frac{2 A}{v_{0}^{1}} \theta+\frac{3 B}{2\left(v_{0}^{1}\right)^{2}} \theta^{2}+\frac{4 C}{3\left(v_{0}^{1}\right)^{3}} \theta^{3}+\ldots\right] \tag{16}
\end{equation*}
$$

where

$$
K=-\left(\frac{2 A}{v_{0}^{1}}+\frac{3}{2\left(v_{0}^{1}\right)^{2}}+\frac{4}{3\left(v_{0}^{1}\right)^{3}}+\cdots\right) \text { and where } \quad 0=\left(a v_{0}^{1}\right) / v_{0}
$$

(ii) The van der Waals adsorption isotherm

The adsorption isotherm for the v.d. Waals equation of state of the adsobed fluid

$$
\begin{equation*}
\left(p^{a}+\frac{\alpha}{\left(v^{a}\right)^{2}}\right)\left(v^{a}-b\right)=R T \tag{17}
\end{equation*}
$$

is derived quite analogously as the virial isotherm on the basis of the equations (13),(15) and (17)

$$
\begin{equation*}
x=\exp [(\phi+\mu \bar{I}) / R T]\left(\frac{v_{0}^{1}}{b}-1\right) \exp \left[\frac{-b}{v_{0}^{I}-b}+\frac{2 \alpha}{R T v_{0}^{1}}\right] \frac{\theta^{\prime}}{1-\theta^{\prime}} \exp \left[\frac{\theta^{\prime}}{1-\theta^{\prime}}-\frac{2 \alpha}{b R T} \theta^{\prime}\right], \tag{18}
\end{equation*}
$$

where $0^{\prime}=(\mathrm{ab}) / \mathrm{V}_{\mathrm{g}}$. A similar but in form somewhat different equation of the isotherm has be8n derived by R.M.Barrer and A.B.Robins (ref. 9).
(iii) Adsorption isotherm of fluid described by the Amagat state equation

It is well known, that the state properties of many 2 D fluids can be dem scribed with the Sohoefield-Rideal equation (ref. 10), which is the 2 D analog of the Amagat state equation

$$
\begin{equation*}
p^{a}\left(v^{a}-v_{b}\right)=i R T \tag{19}
\end{equation*}
$$

where $v_{p}$ and $i$ are constants. $v_{p}$ is the covolume and i refleots the inter:action between adsorbed moleculbs. From (13) and (19) one simply obtains the equation of the isotherm

$$
\begin{equation*}
x=\exp [(\phi+\mu \pi) / R T]\left[\frac{1}{p_{0}^{I}}\left(\frac{i R T a}{V_{0}^{-a v_{b}}}\right)\right]^{i} \exp \left[\frac{i a v_{b}}{v_{o}-a v_{b}}-\frac{p_{o}^{1} v_{b}}{R T}\right] \tag{20}
\end{equation*}
$$

where a is the adsorbed amount.

## THE GENERAL PROPERTIES OF THE ADSORPTION ISOTHERMS

The adsorption isotherms of fluids in homogeneous field of adsorption forces, when expressed as fumotions of the amount adsorbed and of RTInx, can be separated into two parts, one of whioh is the potential energy $\varnothing$ and the other, $f(a)$ depends on the state properties of the adsorbed fluid only, i.e. the general equation can be written

$$
\begin{equation*}
\text { RT1nx }=\varnothing+\mu^{x}+f(a), \tag{21}
\end{equation*}
$$

where for the virial state equation (14) one has

$$
\begin{equation*}
f(a)=\operatorname{RTln}(x / z)=R T \ln 0+R T\left[-\sum_{i=1}^{\infty} \frac{i+1}{i} \frac{A_{i}}{\left(v_{0}^{1}\right)^{i}}+\sum_{i=1}^{\infty} \frac{i+1}{i} \frac{A_{i}}{\left(v_{0}^{I}\right)^{i}} 0^{i}\right] \tag{22}
\end{equation*}
$$

and where $A_{1}=A, A_{2}=B, A_{3}=C \ldots e t c$. For the v.d. Waals equation it follows

$$
\begin{equation*}
f(a)=k+\operatorname{RT} \ln \left(\frac{\theta^{\prime}}{1-\theta^{\prime}}\right)+\operatorname{RT}\left(\frac{\theta^{\prime}}{1-\theta^{\prime}}\right)-\frac{2 \alpha}{b} 0^{\prime}, \tag{23}
\end{equation*}
$$

where

$$
\begin{equation*}
k=\operatorname{RT}\left[\ln \left(\frac{v_{0}^{l}}{b}-1\right)-\frac{b}{v_{0}^{1}-b}\right]+\frac{2 a}{v_{0}^{1}} . \tag{24}
\end{equation*}
$$

For the Amagat state equation (19) describing the gas-like fluid

$$
\begin{equation*}
f(a)=\operatorname{RTln}(x / z)=i R T \ln \left(\frac{i R T a}{V_{0}^{-a v_{b}}} \frac{1}{p_{o}^{1}}\right)+\frac{i R T a V_{b}}{\nabla_{0}^{-a \nabla_{b}}}-p_{o}^{1} \nabla_{b} \tag{25}
\end{equation*}
$$

This is illustrated on the isotherms of krypton adsorbed at temperature $T=180 \mathrm{~K}$ on the zeolites CaA and NaX, shown in Fig. 5.


Fig. 5. The isotherm of krypton at $T=180 \mathrm{~K}$, evaluated by the eqn. (20) for different values of $\eta=$ $\phi_{\text {gin }}$ and $v_{2}=0.325 \mathrm{~cm} / \mathrm{g}, v_{1}=58.0$ om $/$ mole and $i=0.5$. Experimontal isotherms on the zeolite CaA (e) and $\mathrm{NaX}(\mathrm{O})$ are overtaken from the paper of B.P.Bering at all (ref. 6).

When we use the modified Amagate equation ( $\left.p^{a}-p_{m}\right)\left(r^{a}-v_{p}\right)=1 R T$ for the $s$ tate function of liquid-like krypton (ourve 1 in Fier. 3) hafing $P_{m}=-33.6 \mathrm{MPa}$, $\nabla_{b}=58 \mathrm{~cm}^{3} /$ mole and $i=0.5$, the evaluated and experimental isotherms on CaA abe in good agreement, when $\eta=\varnothing+\mu=-7.12 \mathrm{~kJ} /$ mole. For gas-like krypton, the state function of which (curve 2 in Fig. 3) can be described by eqn. (19), $\mu \equiv 0$ and $\eta=-10.95 \mathrm{~kJ} / \mathrm{mole}$, as evident from the Fig. 5.

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