

# ELECTROMAGNETIC INDUCTION IN PLASMA

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## INTRODUCTION

Practical applications of low-temperature plasma have a long history beginning with the use of the electric arc as a source of light or heat. Even the utilization of plasma for chemical process applications began early with the arc process for nitrogen fixation. Studies of the properties of plasma and of elementary processes in plasma, motivated by purely scientific interest, accumulated a wealth of information by the time when plasma was first considered as the working medium for MHD power generation a few decades ago. During the last few years widespread interest developed in the application of plasma for chemical and metallurgical processes utilizing plasma jets, high frequency discharges and high power dispersed discharges besides the electric arc. It is therefore to be expected that this Symposium will be only the first in a sequence of similar symposia.

The plasma property of greatest interest in all these applications is the ion-electron concentration of the plasma and the mobility of the electrons; i.e. the electrical conductivity of the plasma. The various elementary processes affecting this fundamental property are receiving great attention.

In the following paragraphs I have not dealt with this important and well-developed field and concentrated attention on another truly unique property of plasma—its diamagnetic moment and the interaction of the diamagnetic moment with the containing electromagnetic field. Herein it will be possible only to give a short report about the theoretical developments of this subject and to point out the new results.

## DIAMAGNETIC MOMENT OF PLASMA

The ions and electrons of a plasma in a magnetic field circulate around the magnetic lines of force in such a way that the resulting currents tend to reduce the intensity of the magnetic field. The radius of the path of the individual charges is in general so large that quantum limitations become negligible and the laws of classical mechanics apply. One would expect, therefore, that the plasma would exhibit a strong diamagnetic moment and the diamagnetic currents to exchange energy with any electrical vortex field present. But, as has been shown by H. A. Lorentz<sup>1</sup> and by N. Bohr<sup>2</sup>, the diamagnetic moment of a plasma in thermal equilibrium is reduced to zero by the currents which develop at the boundaries of the plasma. Those ions and electrons which reach the boundaries of the plasma are reflected back and form currents circulating in the opposite sense to the charges inside of the plasma, annihilating their magnetic moment.

In contrast to this a plasma column which is not in thermal equilibrium,

for example, a plasma column where the ions and electrons are absorbed at the boundary, does possess a strong diamagnetic moment. This phenomenon was demonstrated experimentally by M. Steenbeck<sup>3</sup> in 1935.

In Gaussian units the resulting magnetic induction has the value given by equation (1)

$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M} \quad (1)$$

where  $\mathbf{H}$  is the magnetic field strength impressed from the outside on the plasma and  $\mathbf{M}$  denotes the magnetic moment per unit volume. If the moment is diamagnetic then  $\mathbf{M}$  is negative and  $\mathbf{B}$  is smaller than  $\mathbf{H}$ .

The diamagnetic moment due to the motion of individual ions and electrons per unit volume of a low density, fully ionized plasma is given by equation (2)

$$\mathbf{M} = -\frac{\mathbf{B}}{B^2} P \quad (2)$$

assuming that collisions can be neglected. Here  $P$  denotes the pressure of the plasma in erg/cm<sup>3</sup>. The above diamagnetic moment of the plasma may be greatly reduced by collisions of the ions and electrons, but in any case it has a definite value for any given plasma pressure and magnetic induction.

Besides the diamagnetic moment caused by the circulating motion of the individual ions and electrons macroscopic vortex currents can generate magnetic moments in a plasma. Such vortex currents may be driven by unbalanced electrical vortex fields, or by unbalanced Lorentz forces resulting from the flow of plasma across a magnetic field possessing vorticity. These vortex currents may be paramagnetic or diamagnetic depending on the sense of the driving forces. The resulting magnetic moment of the plasma can, therefore, assume any value required by the overall system.

### ENERGY EXCHANGE BETWEEN PLASMA AND FIELD

According to the principles of classical electrodynamics the currents which constitute the magnetic moment of the plasma have to transmit

$$-c\mathbf{M} \text{ curl } \mathbf{E} \quad (3)$$

energy per unit volume in unit time to an electrical vortex field. The  $\mathbf{M}$  is the magnetic moment per unit volume, and energy is counted positive when transmitted from the plasma to the field.

Similarly the vorticity of the magnetic moment, which corresponds to a current density,  $c \text{ curl } \mathbf{M}$ , has to transmit

$$-c\mathbf{E} \text{ curl } \mathbf{M} \quad (4)$$

energy to the electric field  $\mathbf{E}$ .

In other substances which exhibit paramagnetic or diamagnetic moments energy exchange between the magnetic moment and the field is prohibited by quantum effects. It is not surprising therefore to find that Maxwell's equations of the electromagnetic field do not contain terms corresponding to the energy exchange modes described by Eq. (3) and Eq. (4). Maxwell's equations summarize the experience gained of ordinary materials excluding

plasma. Full description of the behaviour of plasma in an electromagnetic field requires additional terms in the field equations.

### THE EXTENDED FIELD EQUATIONS

The new terms required in the field equations for the description of plasma have been derived in analogy with the original terms of Maxwell's equations. The validity of the extended field equations is demonstrated by the energy balance equation, which derives from the extended field equations and contains all energy exchange terms operative between plasma and field. For the complete derivation reference is made to another publication<sup>4</sup>; here only the principal results can be described briefly.

The extended equations are equations (5) and (6).

$$\text{curl } \mathbf{B} = \frac{\epsilon}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j} + 4\pi \text{curl } \mathbf{M} \quad (5)$$

$$\text{curl } \mathbf{E} = -\frac{4\pi}{c} \frac{\mathbf{B}}{B^2} \frac{\partial}{\partial t} \left( \frac{\mathbf{H}\mathbf{B}}{8\pi} \right) - 4\pi \frac{\mathbf{B}}{B^2} \mathbf{M} \text{curl } \mathbf{E} \quad (6)$$

The energy equation also shows that in the case of a plasma, where energy is exchanged between the diamagnetic moment and the field, the energy flux vector is defined by equation (7) instead of the usual Poynting vector.

$$\Sigma = \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \quad (7)$$

### VOLTAGE INDUCED BY PLASMA

The new term in the induction law is itself proportional to the vorticity of the electric field. Combining the terms containing curl  $E$  we obtain equation (8).

$$\left( 1 + 4\pi \frac{\mathbf{B}}{B^2} \mathbf{M} \right) \text{curl } \mathbf{E} = -\frac{4\pi}{c} \frac{\mathbf{B}}{B^2} \frac{\partial}{\partial t} \left( \frac{\mathbf{H}\mathbf{B}}{8\pi} \right) \quad (8)$$

If the diamagnetic moment of the plasma is such that

$$\mathbf{B} = -4\pi \mathbf{M} \quad (9)$$

then the expression in the bracket on the left-hand side of Eq. (8) is zero and the equation becomes indeterminate. In such case the vorticity of the electric field, measured in a frame of reference at rest relative to the magnetic field, may have any value and may persist indefinitely, independently of the rate of change of the magnetic flux.

In other words, if the criterion stated by Eq. (9) is fulfilled, then the field equations permit the generation of steady state-induced voltages, but the field equations alone are not sufficient to determine the magnitude of the induced voltage. The existence of an electrical vortex field in a magnetic field implies inductive energy transfer. The magnitude of the vorticity of the electric field induced by the plasma is therefore determined by the energy available for transfer between the plasma and the field.

This result should not be surprising. There is a well known analogous

case where the field equations are not sufficient for the determination of the generated voltage; the voltage of an electrochemical cell is determined by the free energy release by the chemical reaction in the cell.

## INDUCTIVE ENERGY TRANSFER BETWEEN PLASMA AND FIELD

The energy transmitted from the plasma to the field may originate from the expansion work of plasma flowing across a magnetic field. In this case the thermal energy of plasma is converted into free energy which is then transmitted to the outside electrical circuit by induction at constant magnetic field. The induced voltage is proportional to the flow rate of the plasma and the expansion work per unit mass of the plasma.

The process may be reversed and plasma may be compressed in continuous flow by energy transmitted by induction from the field to the plasma. This process is inherently connected with substantial energy losses.

It also appears possible to heat a non-flowing plasma by inductive power transfer from the field. In this case electrical energy (free energy in the thermodynamic sense) is converted into thermal energy.

In all cases of inductive power transfer the existence of a steady state induction phenomenon requires the fulfilment of the criterion stated by Eq. (9). This requirement can be satisfied in general only by the excitation of paramagnetic and diamagnetic vortex currents in the plasma. Energy balance considerations indicate that such vortex currents may be excited automatically with the required intensity, but as yet there is no experimental evidence available to prove this expectation. Truly the criterion stated by Eq. (9) forms the central problem for future research in this subject. Should experiments confirm the theoretical predictions then it is likely that the new induction phenomenon will have great effect on the future development of generation of electrical power. It will also provide a new method for the heating of plasma for process applications.

## SUMMARY

Consideration of the diamagnetic properties of plasma leads to the prediction of a new phenomenon, the possibility of steady-state inductive power transfer between plasma and the enclosed electromagnetic field. Although the full explanation of this phenomenon will require further study and experiment, it appears reasonable to expect that this induction phenomenon will have a significant effect on the future development of the generation and distribution of electric power. It is believed that this subject is of interest to the participants of this Symposium partly because of the effect it may have on the future availability of electrical power for large scale process applications, and partly because of the possibilities it offers for the generation and heating of low-temperature plasma.

## References

- <sup>1</sup> H. A. Lorenz. Göttinger Vortrag (1914).
- <sup>2</sup> N. Bohr. *Dissertation*. Copenhagen (1911).
- <sup>3</sup> M. Steenbeck. *Wiss. Veröff. Siemens-Konz.* **15** (2), 1 (1936).
- <sup>4</sup> B. Karlovitz. *Equations of the Electromagnetic Field Containing Plasma; Inductive Power Transfer in Time Invariant Fields*. In the press.